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On Continuity: Rush Rhees on Outer and Inner Surfaces of Bodies

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Abstract

This article presents an edited excerpt from a hitherto unknown fragmentary treatise by Rush Rhees. In the treatise, Rhees gives his account of the problem of continuity that he had started elaborating before he became acquainted with Wittgenstein. The excerpt, which contains Rhees' original distinction between outer and inner surfaces of bodies, builds on Brentano's theory of the continuum and his doctrine of plerosis. This treatment of continuity sheds light on Rhees' early philosophical development and confirms that even though he and Wittgenstein discussed the problem of continuity, Rhees' own approach remained distinct from that of Wittgenstein.

Introduction

Readers of Rush Rhees' *Discussions of Wittgenstein* will remember the essay "On Continuity: Wittgenstein's Ideas, 1938." In the preface to the book, Rhees describes the origins of the essay:

> The essay "On Continuity" has not been printed before and I wondered if I should include it. The ideas are Wittgenstein's, and so are the chief examples and phrases. The first half says little that cannot be found in Wittgenstein's own words in the *Remarks on the Foundations of Mathematics*. But it goes with the second half which discusses questions ("continuity outside mathematics") on which he did not write after 1930. I had been trying to write on continuity in 1938, and in August he offered to talk about it with me. For about three weeks we met every day. Once Francis Skinner was present; no one else was. The discussions were in the afternoon and they were long (one finished after seven hours). In the last few of them I took down some things while he was speaking. In the evenings or next day I wrote down what I had

> understood. I have written from these notes, trying at various points to work out what he meant.[1]

In the following, we present an excerpt from what we assume is the writing on continuity that Rhees mentions in this passage. The excerpt is from a 77-page typescript found among the papers of Alfred Kastil that are in the Franz-Brentano-Archives at the University of Graz. The typescript is divided into two chapters written in well-shaped prose, thus appearing as the elaborated beginning of a longer treatise. Our excerpt represents the core of chapter 2.

In chapter 1, Rhees explicates what kind of continuity he will be concerned with in his investigation. Most importantly, he excludes the mathematical concept of continuity. His focus, he says, will instead be on continuity in immediate experience. Thus, the scope of Rhees' account may be related to the second part of the essay "On Continuity: Wittgenstein's Ideas, 1938." The most striking feature of the excerpt, however, is how it differs in general from the essay in *Discussions of Wittgenstein*. As the excerpt shows, Rhees built on Franz Brentano's theory of the continuum and his doctrine of plerosis in order to clarify the then-current philosophical discussion of the problem of continuity. This ambition in philosophical scholarship stems from a time before Rhees became acquainted with Wittgenstein, and the theoretical approach differs from that taken in his discussions with Wittgenstein later on. A note in a letter to Brian McGuinness, in fact, suggests that Wittgenstein was not familiar with Brentano's ideas:

> I came to know Wittgenstein after I had known Kraus and studied Brentano, and I was interested to learn if I could whether Wittgenstein had read Brentano. I think it is certain that he had not. [. . .] I often mentioned Brentano's views to him, and he never showed the slightest sign that he was acquainted with them. [. . .] And I cannot really find anything in Wittgenstein which reminds me of Brentano.[2]

Rhees' first job after completing his degree in philosophy at the University of Edinburgh was a temporary teaching post at the University of Manchester.[3] That post expired in 1932. During the subsequent years, he spent several long periods in Innsbruck studying with Prof. Alfred Kastil, a former student of Brentano and Anton Marty. Some 15 years before Rhees first went to Innsbruck, Kastil had been appointed editor of

1. Rhees (1970: vii); underlining by Christian Erbacher. Wittgenstein wrote on questions connected with "continuity outside mathematics" in his so-called phenomenological phase in 1929.
2. Letter from Rhees to McGuinness, dated 6 May 1963, kept at the Richard Burton Archives at the University of Swansea, UNI/SU/PC/1/1/3/5.
3. Cf. Phillips (2006).

Brentano's papers. At the time of Rhees' visit, Kastil was editing the book *Kategorienlehre* containing Brentano's theory of the continuum.[4] Rhees developed a keen interest in Brentano's views and later even made suggestions for Kastil's edition. Kastil pointed out that he regarded Rhees as capable of working out Brentano's fragmentary theory:

> The principal aim of his visit, and the one which he chiefly followed, was to make himself acquainted with the philosophy of Franz Brentano and with that of the most important of Brentano's pupils, Anton Marty. Since then he has deepened his knowledge of these philosophers that his understanding of their position is not inferior to that of their immediate pupils. [. . .]
>
> He showed at once how well he had developed the fruits of his study in Innsbruck. He was in a position to suggest many improvements to the latest volume of Brentano's posthumously published works, *Kategorienlehre*, of which I shall make use in a new edition.
>
> Among the problems which stand in the forefront of philosophical interest today, the problem of the nature of the so-called relations offers particular difficulties. Brentano in his enquiries pointed out new paths in this connection, although he did not follow them out completely. Mr Rhees has undertaken, with a decided probability of success, I think, to carry the work further in this field. He laid before me the plan of a theory of relations which we discussed, particularly that part of it which had to do with the so-called relations of comparison, as we met one another daily during his summer visit.
>
> At Christmas time and during the past 5 weeks, we have been busy with the analysis of the relations of continuity. Brentano struggled with the difficulties of the continuum repeatedly throughout his life and developed a general theory of continuity, which sets forth the general laws for continua of various numbers of dimensions. In this connection, he did not neglect the peculiarities which distinguish topic continua from chronic continua. But Brentano devoted special study to double continua, of which motion is the most important example. Here, he developed the conceptions of teleiosis and of plerosis, the former of which applies to differences of velocity, the latter to the quantitative differences in the connections of boundaries. But even in regard to these relations of continuity, Brentano's theory remains incomplete; indeed he indicated to me a few days before his death that his theory was in a process of alteration, without giving any further indication of the kind of improvements which he had in mind. It now appears very probable that Mr Rhees's acumen and unsparing diligence have succeded [sic] in finding the proper approach here.[5]

When Kastil wrote these lines, Rhees was a doctoral student at Cambridge. G. E. Moore was his supervisor and had suggested he attend Wittgenstein's classes. Although Rhees followed this advice, he was ini-

4. cf. Brentano (1968).

5. Presumably a letter of recommendation, dated Easter 1935, kept at the Richard Burton Archives at the University of Swansea, UNI/SU/PC/1/1/2/3.

tially sceptical about Wittgenstein's philosophising and wanted to devote his studies to the philosophy he had learned in Innsbruck. In November 1933, Rhees wrote to Kastil:

> I went to Wittgenstein a few times. He very much gives me the impression of being a straightforward and honest person; however, I don't think that I will go to him more often. I did not make this decision instantly, as Moore seems to be very appreciative of Wittgenstein. I, in turn, value Moore's judgment very highly, and I know that he would not have his opinion without a reason. Nonetheless, I think I will not go anymore. I find his style of lecturing confusing. He never prepares – as when he does, his lectures suffer. (I am convinced that he is no posturer in this, although he is probably mistaken.) He continuously speaks in similes (which are only partly actual examples), and says about himself that he always thinks in similes. If something does not become clear, he does not try to give an explanation in simple words but instead looks for a new simile. This method, though, is in accordance with his philosophical position, according to which the answers to the most important philosophical questions cannot be given through propositions or theories, but can only be "shown" by means of similes "or symbolic forms." Therefore, he says that he may be the right man for philosophy. (This is again, I believe, only naivety, not a sign of vanity.) But this is why his lectures do not show a clear thread. Currently he lectures on the philosophy of language, particularly on the idea of meaning. He constantly emphasizes that the matter is exceptionally difficult. Sometimes he grabs his head, giving the explanation, "All this is tremendously difficult, we are in the middle of hell right now." And I asked myself if some of the attendees have any clear impression about philosophy, barring that the whole (quite undefined) matter is "tremendously difficult." This I regard as pedagogically bad. I hear that only after having heard him [Wittgenstein] for a fairly long time one starts to recognize how much one gets from him. That I am willing to believe. But life is short; and the question is whether I would not profit even more if I used the time for something else (e.g., for studying Marty's works). And at the moment, it seems to me that this question has to be answered in the affirmative.[6]

Although Rhees later changed his mind and regularly participated in Wittgenstein's lectures, he continued working, in Brentano's vein, on the problem of continuity.[7] In 1936, however, Rhees abandoned his plan to finish a dissertation. He once more visited Kastil and then substituted for his former professor at the University of Manchester, teaching a wide range of topics in philosophy. Wittgenstein travelled back and forth between Vienna, Cambridge and Norway during these years. When he decided to stay in England and resumed lecturing in spring 1938, Rhees

6. Letter Rhees to Kastil, dated 5 November 1933, kept at the Franz Brentano-Archiv, Nachlass Alfred Kastil, catalogue number K.1.96.1, inventory number 000.616-000.622; the original is written in German, translated by Tina Schirmer and Christian Erbacher.
7. Cf. Erbacher (2016).

was among the hand-picked invitees. This renewed their relationship and led to a phase of collaboration. Wittgenstein asked Rhees to translate the book that he planned to publish with Cambridge University Press, and he invited him to discuss "continuity." Wittgenstein knew Rhees was trying to write on this topic and encouraged him:

> As you know, I wish you lots of luck with your writing. Just stick to it; and if possible, sacrifice coherence sometimes. I mean, if you feel you could just now say something, but it isn't exactly the thing which ought to come in this place – rather say it and jump about it a bit than stick to the "single track" and not get on. That is, if you can do it. If you can't jump, just plod on.[8]

Rhees thought about submitting his writing as an application for a fellowship. Wittgenstein supported this idea, but eventually learned that Rhees decided against it:

> I found your first chapter here and was disappointed that you had not sent it in. I think it was wrong not to do it and I think you ought *still* to do it if there is a chance that it might be overlooked that you're a bit late. I have only glanced at a few pages & can't do more at present, but I didn't at all have a bad impression! So why the hell you should wish to be your own examiner I can't see.[9]

We assume that the "first chapter" Wittgenstein is referring to in this letter is the first chapter of Rhees' treatise from which our excerpt is taken. As noted above, the first chapter is concerned with defining the scope of the study. The second chapter begins with a discussion of positions by prominent Cambridge scholars, in particular William E. Johnson (referring to *Logic*, Johnson 1921–24), Bertrand Russell (referring to *Introduction to Mathematical Philosophy*, Russell 1919) and Alfred N. Whitehead (referring to *An Enquiry Concerning the Principles of Natural Knowledge*, Whitehead 1919). This sets the stage for Rhees' own and contrasting account that builds on Brentano.

The following excerpt is the core of Rhees' attempt to formulate his original idea of integral surfaces of bodies. Although one may notice traces of Rhees' encounter with Wittgenstein, the theoretical proposal and the dense, jargon-free prose show the genuine approach and style of the young Rhees. The excerpt may, therefore, illuminate Rhees' early philosophical development and his philosophising as being distinct from his discussions with Wittgenstein. The different vein of the investigation also supports what Rhees stated in his introduction to "Wittgenstein's Ideas, 1938," namely,

8. Letter from Wittgenstein to Rhees, 9 September 1938, cf. Wittgenstein (2012: 281, letter 230).

9. Letter from Wittgenstein to Rhees, 2 October 1938, cf. Wittgenstein (2012: 285, letter 233).

that they indeed show Wittgenstein's treatment of the topic. Rhees presents his own account of continuity in the here-edited excerpt of his treatise.

Acknowledgements

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Editorial note

The original document of Rhees' fragmentary treatise consists of 77 pages of typescript divided into two chapters. The chosen excerpt is from the second chapter, namely the pages 35–66 in Rhees' own pagination. On approximately one-third of these pages, there are occasional handwritten annotations which are underlinings, interlinear notes or comments in the margins. These annotations were made by Alfred Kastil and are partly written in short hand. The legible ones are either German translations of single expressions or, when they appear in the margins of pages, mainly content-related remarks or sketches.

The here-edited excerpt presents the text typewritten by Rhees. Every page break in the original document is marked with a vertical bar (|). The raised numbers to the right of the bar denote the following page and represent the page numbering Rhees used in the original document. We include this page numbering to enable direct referencing of Rhees' text. His paragraphs and indented lines have been retained. Passages are underlined only if they are marked thus in the typewritten text and can hence be distinctly ascribed to Rhees. The footnotes inserted in the original document are presented at the bottom of each page of the edited text using continuous numbers. Very few obvious errors in spelling and punctuation have been corrected; to indicate where corrections have been made, they are placed in chevrons (‹ ›). Corrections have only been annotated if they cause a change in semantics.

A complete edition of Rhees' fragmentary treatise will appear in a forthcoming book on the editorial work of Wittgenstein's literary executors.

Outer and Inner Surfaces of Bodies

Excerpt from a fragmentary treatise by Rush Rhees[⋆]

We probably should have no conception of a surface if we did not have volumes of certain limited sizes given in immediate experience. But

⋆. Rhees' treatise fragment is included in the holdings of the Franz Brentano-Archiv, Institut für Philosophie, Karl-Franzens-Universität Graz, Nachlass Alfred Kastil; catalogue number S.6.24.06, inventory number 016401-01677.

when we say that volumes are given as of limited sizes we imply that they are given as having ends. And we may be led to speak about the end of a body (or its tip, its edge, or its side, or its top or its bottom) in some given direction from its centre. It may of course be suggested that when we speak in such connections about the body's "having" an end or side it is only the syntactical figure which suggests that the side is a constituent element of the body; and that every statement which appears to be about "the side" is really a statement about the whole body and its relation to its surroundings. But such a view seems to me both unnecessary and inadequate. I say that it is unnecessary because it seems to me that it would be perfectly sensible to speak about "this side or surface of the body" in referring to a constituent element of it. There are various considerations which suggest, | [36] at any rate, that we mean by expressions such as "surface", "edge", and "beginning" to refer to something not identical with the whole of what is said to have them.

In the first place, surfaces of bodies do seem to have special characters of their own and to be continuous wholes of a different sort from the bodies to which they belong. Surfaces are frequently spoken of as being continuous in only two dimensions, for instance, and they are compared in this respect with bodies and volumes which are continuous in three dimensions. We seem in such cases to be talking about a surface and not about its three dimensional body. We seem no less clearly to be doing so when we compare different surfaces with one another.[1]

Further, when we say that two surfaces in different planes intersect to form an edge of the body, and that this edge is a line which is one dimensional, we do not seem to be saying merely something about the relation of the two surfaces to one another. What we say about the edge no doubt entails a statement about the relations of the two surfaces. But we also ascribe positive characters to the edge itself. And in doing so we imply that it is something, although it would obviously be absurd to suggest that it could be separated from that of which it is the edge.

Again, we may say that the beginning boundary or "beginning end" of a temporal process has a character as boundary (or end) which is different from that of the ending boundary of the process just because each is boundary in a different temporal direction; or in other words, just because the one is the beginning and the other is the end. So that here once more we ascribe positive characters to ends as such without making an assertion about the relations between that which has them and | [37] something outside it. There is, I believe, an analogous distinction in character between opposite sides or outer surfaces of bodies in space,

1. I shall point out later certain other differences between surfaces and bodies as continuous wholes.

although here the difference is not so striking because it always seems obvious that the body might be turned about, so that the side which now bounds in one direction might bound in the opposite one. However this may be, the distinction between the different temporal boundaries seems to me to be plain enough and to be sufficient for the point I want to make.

What we mean by an end or a boundary or a side, in this sense, can best be brought out, however, by considerations of the following kind. Suppose that we are concerned with some particular flat end or side of a given body. Suppose then that the body be divided through the centre parallel to the side in question; and then that the half of which this side is still a side be halved again in the same way, and that the half of this half which still has that side be halved again, and so on, without an end to the process of halving the part of which this side is a side. It is clear that what we mean by "this side" could not possibly be affected by this series of divisions (on the assumption, of course, that the halving produces no other changes in the volumes concerned). It is equally clear that if we take "part" to mean "part which has some volume", then there is no part which would be unaffected in the course of an endless process of divisions of this kind. Consequently it appears that what we mean by "the side of the body" is not any part of the body in this sense. Yet we do mean something that is actually given and that would be unaffected throughout such a division.

It is clear that such a side or surface would always be connected with, in the sense of being the side of, some part or other. It is not even logically possible that the side should be separated from any and every part having any volume. | [38] But there is no part having any particular volume of which it must be the side or surface; and this amounts to saying that there is no particular part of which it must be the side. This does not mean, of course, that the side is the side of a "universal", or that it might be the side of any one of a variety of parts which were quite different individuals or which could exist simultaneously in separation from one another. What it means is that there is no part having any volume of which it could be said truly, "It would be absurd to speak of this side without this part". It is for this reason that the consideration or supposition of a division of this kind is useful in bringing out what is meant by side or surface of a body. For the surface or side thus appears as something which must remain throughout such a division what it essentially is, namely the side of a body. It is thus clearly given as something essentially relative which cannot without obvious absurdity be thought of as having a separate existence. And yet the fact that we can think of the side as something particular and definite without thinking of any definite

volume of which it is essentially the side makes clear the distinction between the thought of the side and the thought of what is bounded by it.[2]

Perhaps it will be objected, however, that I have not done enough to show that anything of the sort which I am supposing to remain unaltered throughout such a division is ever actually given. Certainly something is given which is endlessly divisible in the way described. But it may be said that all that this shows is that such a process of divisions can never reach a smallest possible volume; and so that the possibility of such an endless division goes no way to show that anything other than parts having volumes is given. | [39]

If the objection said no more than this I should reply simply that it does not explain what can be meant by the st‹a›tement (which I suppose is admitted to be true) that what we first call the side of the body remains the same throughout such a process of divisions, no matter how far the process be carried. But it may be said that we can explain what is meant by this if we treat the side of a body as an abstractive class of volumes adjoining another body; if, in other words, we give an account of the boundaries of bodies analogous to that which Whitehead has given of the boundaries of "events" in his Principles of Natural Knowledge. Such an account is entirely in terms of relations of extension between volumes, and does not require anything like the integral surfaces of which I am speaking.

But even if the application to bodies of Whitehead's analysis of "events" were otherwise entirely unobjectionable, I do not think it would give a satisfactory account of what I have been calling the side of a body. The most relevant passage[3] in Whitehead's theory starts with the notions of "intersection" and of "dissection" of events. "Two events 'intersect' when they have parts in common... Events which do not intersect are said to be 'separated'... A 'dissection' of an event is a separated set such that the set of intersectors of its members is identical with the set of intersectors of the event." In terms of these notions he defines what he means by "junction of events". "Two events x and y are said to be 'joined' when there is a third event z such that (i) z intersects both x and y, and (ii) there is a dissection of z of which each member is a part of x, or of y, or of both". Junction thus includes intersection, but it also includes "adjunction", in which case the | [40] joined events are "separated", i. e., have no parts in common. With the help of adjunction Whitehead can then define "injunction" as well. "An event x is said to 'injoin' an event y when (i) x extends over y, and (ii) there is a third event z which is separated from x and adjoined to y."

2. See Brentano, "Vom sinnlichen und noetischen Bewusstsein", p. 95.
3. Opus cit., p. 102.

Suppose that in this definition of "injunction" we substitute "body" for "event", and suppose that we take x and z to be a red block and a blue block respectively. Suppose further that the red block and blue block are in contact with (or adjoin) one another. There is then a part of the red block which is in contact with the blue block but is not identical with the whole red block. This part corresponds to what is called "event y" in Whitehead's definition, and we might say that the red block "injoins" this part of itself. If we call this injoined part a, then there is also a part b that is injoined by a; and also a part c that is injoined by b, and a part d that is injoined by c, and so on. It is clear that this relation of injunction is what Russell would call an asymmetrical transitive relation, so that whatever injoins a part injoins whatever is injoined by that part. Any series of parts injoined by the red block in its contact with the blue block thus resembles the series of volumes involved in the process of divisions of parts nearest the end of a body which I have described above. But it is important to notice that in the present account the relation of injoined to injoining parts is not specified by saying (inter alia) that they are all parts of which this particular side is a side, but rather by saying that they are all parts which are in contact with or are adjoined by the blue block.

It might now be said that a series of parts injoined by the red block in its contact with the blue one form an abstractive class, and that such an abstractive class defines | [41] anything that can reasonably be meant by speaking of the side or surface of the body. "The properties of an abstractive class", according to Whitehead,[4] "secure that its members form a series in which the predecessors extend over their successors, and that the extension of the members of the series (as we pass towards the 'converging end' comprising the smaller members) diminishes without limit... Any property of the individual events which survives throughout members of the series as we pass towards the converging end is a property belonging to an ideal simplicity which is beyond that of any one assignable event... The series itself is a route of approximation towards an ideal simplicity of 'content'." The series of injoined parts might thus be regarded as a route of approximation by reference to which we define that ideal simplicity of content which is what is meant when we speak of "the side or surface of the red block that is in contact with the blue block." It would then be clear that what is spoken of as the side or surface of the block is really a fiction and not a real constituent element of it.

Now the question whether such an account really does explain what is meant by the side of a body without assuming that there is anything

4. Opus cit., p. 104.

not a volume which is a constituent element of the body depends above all on the question whether what is said in it about "adjunction" is an adequate account of what we mean by contact of bodies. Adjunction is defined in terms of intersection and separation. But what can be meant by "intersection" in the case of two adjoining bodies? If a blue block adjoins a red block I cannot see any sense in speaking of a third body which has parts in common both with the blue block and the red block, and which has no part which is not also a part of the blue block, or of the red block, or of both. And consequently I do not see how a reference to intersection can be used to make clear what we mean by the contact | [42] of the two blocks. It is equally obvious that "separateness" or "having no parts in common" is not enough for contact, although it is necessary. And I see no way of saying what is meant by contact or "adjunction" of the bodies without referring to the side of the red block and the side of the blue one. The two sides or surfaces must touch. But if we do put the matter in this way, then our procedure is really the reverse of a method like Whitehead's. For according to that method we must know what is meant by adjunction before we can say what is meant by a side; whereas what I am maintaining is that in the case of bodies we must understand what a side is before we can know what is meant by adjunction or contact. And if I am right it will follow that we cannot understand what is meant by the "series of injoined parts" without assuming that these parts all have the same side; that the side must be given if "injunction" is to be possible at all, and that the side cannot be identified with any "injoined" part of the series. It follows also that the side or surface cannot be treated as an "ideal simplicity of content" towards which the series of injoined volumes is a "route of approximation". What I mean, and what I think is ordinarily meant, by a side or surface cannot conceivably be a property of any volume, and it cannot be the ideal limit of a series of volumes. A side or surface has properties which volumes do not have, and perhaps there is some sense or other in which it could be said to have a "simplicity" which volumes do not have. But it certainly cannot be called "an ideal simplicity of volume", or anything similar to it. Further, an ideal simplicity of content towards which a series of injoined volumes "converged" or "approximated" would not have that essential characteristic of a side o‹r› surface which we indicate when we say that it is always a side or surface <u>of</u> something. It is clear that this does not mean that the side is "injoined" by anything, since only that can be injoined which <u>has</u> a side or surface. | [43]

Perhaps it will be said, however, that there is a sense in which "adjunction" of bodies can be defined simply in terms of intersection and separateness of volumes, even though it is impossible to speak of a third <u>body</u> as intersecting two separated bodies. It might be said that there is

always a spatial volume which intersects the volumes of the two adjoined bodies, and which is such that every part of this volume is a part of the volume of the one body, or a part of the volume of the other, or of both. We are thus able, it may be suggested, to give an account of adjunction of bodies simply in terms of the relations of their volumes to one another, without the assumption of sides or surfaces in the sense in which I have been speaking of them. Or perhaps the matter could be put more clearly by saying that there is a place which intersects the places of the two bodies, and that there is a dissection of this place such that each member is a part of the place of the one body, or of the other, or of both.

If this suggestion removes any difficulties that may be felt about speaking of intersection in connection with adjoining bodies, it seems to me to raise new difficulties as to what is meant by the separateness of adjoining bodies. By definition whatever adjoins anything else is also separate from it. But if the suggestion of the previous paragraph is right, then the blue block could not be said to adjoin the red block in any different sense from that in which the blue block plus one third of the red block could be said to adjoin the other two thirds of the red block. And of course one could talk about the side of any part within the red block in exactly the same sense as one can talk about the side of the red block which is in contact with the blue one. If we do not accept these consequences, – and they seem very strange to me, – then I think we must give up the attempt to describe contact of bodies simply in terms of relations of intersection among volumes within the space which they jointly fill. And I think we must | [44] agree that it is necessary to know what is meant by the side of the one body and the side of the other before we can understand what is meant by their contact.

I say that the consequences mentioned seem strange to me partly because it seems to me that when we speak of two different bodies that are in contact we are not speaking simply of adjoining areas of the region which they occupy together. When I say "a match box is in contact with this desk" I do not mean simply "there is a spatial region adjoining the region occupied by the desk". I mean, among other things, that one side of the match box touches the desk top. It may be suggested that the question of whether that which fills a certain area is a single body or only a part of one is a question which cannot be decided simply by reference to relations of continuity; that we say that the match box and the desk are different bodies because they have a certain mechanical independence of one another, and not because their adjunction is any different from the adjunction of parts of a single body. On this point I shall have something to say later on. My contention at present is not that we say that the match box and the desk are different bodies because their

contact is different from the adjunction of parts within any one of them. I am contending simply that we do regard them as different bodies, whatever our reasons may be, and that we do mean something by the contact of different bodies which cannot be expressed simply in terms of intersecting volumes. For it seems to me that we should not be able to speak sensibly about distinct bodies at all unless we did recognise that they have definite sides or outer surfaces in a sense in which their internal parts do not.

I recognise that I have not said anything which proves this. There seems to me to be some obvious discontinuity between two bodies in contact which is not to be found between | [45] the parts of any one body, or between adjoining regions of an otherwise undifferentiated area. I hope that my view on the matter may seem more plausible when I have said what I want to say about "inner surfaces" in distinction from outer surfaces or sides of bodies. But nothing I shall say on this point would be conclusive against anyone who denied from the outset any difference between the contact of different bodies and the adjoining of different regions within a larger space.

The method which I should employ in bringing out the notion of an inner surface is analogous to that which I have given for the clarification of the notion of an external surface. And I want to maintain with reference to inner surfaces as well that they are not fictions useful for expressing relations between volumes, but are integral constituents of bodies. Suppose that we take any body which has two parallel sides, and imagine it divided into thirds by two divisions, each parallel to one of the sides in question. Suppose then that we take the central one of these three parts and divide it similarly into thirds, and so again with the central one of the three parts of it. We can think of a series of divisions of this sort as being prolonged without limit. We might speak of such a series as one which constantly "closed in" towards the "middle" of the original body. If the series be thought of as unending, then it is clear that at the middle there is no part having any volume which could not possibly be affected by such a series of divisions. It is clear also that the middle of every volume concerned in such a series of divisions would be identically the same, and that no matter how far the series of divisions be thought to be carried this middle could not possibly be reached or affected. We have thus the conception of a middle or midway surface as something which is not itself a part having volume, but is nevertheless inconceivable except as included in a part having some volume | [46] or other. It would not be anything at all unless it were a middle or midway surface of something. But there is no part of any particular volume that must include it. In other words, there is no part having any volume of which it could be truly said, "It would be absurd to speak of the middle here

without this part". We can thus think of the middle or midway surface as something particular and definite without thinkin‹g› of a part of any particular volume in which it is included; and in this way we may be helped to distinguish the thought of the midway surface from the thought of anything which includes it, although we cannot think of it except as included in something or other.

It seems to me plain that the thought of such a midway surface is not the thought of an "ideal simplicity of content" towards which the series of volumes concerned could be said to form a "route of approximation", or towards which as a limit the series of volumes could be said to "converge". In the account I have just given the midway surface is not thought of as a limit of a series at all, and particularly not as the limit of a series of volumes. And it is not necessary to think of any limit towards which the series of volumes "converges" in order to have a clear notion of a midway or inner surface[5] included in them.

Such inner surfaces must, I think, be given wherever anything is given as endlessly divisible in this way. And to say that they are given implies that they are not fictions useful for expressing certain relations of volumes to one another, but are integral to what is so divisible. The recognition of them seems to be indispensable to an analysis of what | [47] is meant by saying that one and the same thing is endlessly divisible in this way, just because when we think of the series of divisions as proceeding without limit we do think of something given in what is divided which cannot possibly be affected by it. In so far the notion of an inner surface is analogous to that of an outer surface. The important difference between the two lies in the fact that an inner surface is given as something which must remain unaltered in the course of a double series of divisions, by which the divided volume is reduced simultaneously from both sides. This makes it clear that the surface so given cannot be the outer edge or surface of any part having volume, and that it is logically inseparable from some volume or other on each side of it. Only there is, of course, no specific magnitude of volume from which it is inseparable in order that it should be the inner surface which it is; which is part of what is meant by saying that it cannot be affected in its character of an inner surface through an infinite reduction of the volume which encloses it.

Now I want to suggest that outer surfaces are surfaces only of single and separate bodies, and that no outer surface is found within a single

5. I recognise that the expression "inner surface" is rather barbarous, and that the expression "outer surface" would in ordinary speech be pleonastic. But an expression such as "inner plane" seems to be even less suited to my purposes. And I shall use "inner surface" in the hope that what I have said on the subject makes tolerably clear what I am talking about.

body. Within a single body there are, I suggest, only inner surfaces. Now no inner surface can be integral to, or be a surface of any part of a body without being included in it. In other words, it cannot be a surface of any part which extends no further than it. And if there are no surfaces but inner surfaces within a single body, then there is no surface within the body with which one part of the body might be said to begin and at which another part of the body might be said to end. An inner surface cannot be a surface with which a part begins, since it can be only a surface of something which includes it.

This consequence may seem at first sight rather startling; and I will try to make clearer what it means by considering some objections that may be raised to it. In the first place | [48] it may be said that we can specify precise lines of division between parts of a single body, and can speak both truly and sensibly of parts on opposite sides of these lines which extend just up to them and no further. If a razor edge be placed across a strip of paper we can speak about the part to the right of the razor edge and the part to the left of the razor edge, and we can say that these parts meet at the line which the razor edge touches. It is only saying the same thing in other words, apparently, if we say that the one part begins there and the other part ends there (or that they both begin there or both end there).

In the second place, we may often say truly that one single body is larger than another, – that one strip of paper, say, is larger than another strip. And one of the things that this implies is that the larger body contains a part exactly equal to the smaller body. If we take a simple case such as that of one square piece of paper which is larger than another square piece of paper, for instance, we seem to be justified in saying that the larger piece contains a part having exactly the same dimensions as the smaller piece. And this seems to mean that the larger piece contains a part which has definite boundaries beyond which it does not extend, or at which it "ends".

In the third place it may be said that a single body, such as a piece of paper, may be split up into a number of discrete parts, each of which has definite boundaries. No such part need have acquired or lost anything through its separation from the others. And it seems therefore as though each part must have existed before the separation as a part having just the extent and dimensions which it has now. Each part, that is, must have extended up to what is now its external surface and no further. | [49]

Finally it may be suggested that even according to my own account of inner surfaces there must be a part having some volume on either side of an inner surface, and the part on the one side cannot be identical with the volume on the other. This would seem to mean that the surface definitely divides one part from another. Must I not, in fact, admit either

that a part can extend up to an inner surface and no further, or else that surfaces are fictions and not really to be found within bodies at all?

Such objections might be taken to show that the distinction between inner and outer surfaces which I have been trying to make is unsound, and that it can do nothing to clarify the supposed distinction between continuous parts and bodies in contact. I shall try to show, however, that in so far as what they say is sound they are not objections to the point I am trying to make.

I will begin with the last of them. An inner surface cannot without misleading be said to "have a volume on either side of it" or to be "the place where two volumes meet". If a surface is said to be something up to which two different volumes extend this suggests that the surface itself is not integral to either of them. And I should agree to call a surface in this sense a fiction. No doubt we often do speak of surfaces within volumes in this sense. But then "surface" means much the same as "division" or "demarcation" or "cut" or "partition"; and these words are all abstracta which do not stand for any integral element in what is given or thought of. I have said that an inner surface is inseparable from some volume or other on each side of it; which I take to be just another way of saying that it is necessarily included in some volume or other. But this does not mean that it is "between two volumes" in the sense in which something not identical with and not a constituent of either of two volumes might be | [50] said to be between them. I say that an inner surface is something which itself has no volume but is an element in the continuity of a volume which is on either side of it. If we speak of <u>two</u> volumes, one on either side of it, then evidently an integral surface is integral either to one of these volumes only or to both. But if it were integral to one of them only it would not be an inner surface at all, but rather an outer surface or side. And it would not be anything which could be suggested by the simultaneous series of reductions of a volume from both sides which I have described. If it is integral to both volumes, then there is no volume with an outer surface or side, and there seems to be hardly any justification for speaking of two volumes. There is no volume which has this surface as its beginning or its end; for if it did, the surface would not belong to or be integral to anything in the other direction. For this reason I prefer to speak of an inner surface as an inner surface <u>of</u> a volume, and neither as a surface between two volumes nor as a surface which has "a volume" on the one side and "a volume" on the other.

It may still be said, as in the first of the objections mentioned above, that if a razor edge were placed across a body we could recognise a sharp distinction between one part of the body which was to the right of it and the other part of the body which was to the left of it. And it might be suggested that the razor edge must fall on a line which is the end of a

certain inner surface extending at right angles to the outer surface on which the razor edge is placed. Does not the distinction of the part to the right of the razor edge and the part to the left of it then imply that the parts of the body so referred to are respectively parts to the right and to the left of the inner surface in question? If so then the inner surface must be said to divide parts which meet at it.

I think it is obvious that the "surface" which is here spoken of as defined by reference to the position of the razor | [51] edge is really just a surface in the sense of a "partition", that is, that it is a fiction of the sort already noted. And it is necessary to try to see more clearly what the relation is between statements about such partitions which are said to fall between different parts of the total volume of a body on the one hand, and statements about real connections and real boundaries on the other. I agree of course that when a razor edge is placed across a piece of paper we speak of "all to the right of the line on which the razor edge falls" and of "all to the left of the line on which the razor edge falls." But this does not imply that in such a case we have to do with two <u>things</u>, one of which is to the left of the razor edge and the other to the right. And it cannot be argued that "the part to the right of the razor edge" is something which ends with a surface there in the sense in which a body in contact with another body there would do so. It seems to me that the statement, "There is a surface, defined by reference to the razor edge, at which one part begins and the other ends" is similar in certain important respects to statements such as "There is a dissection of the event z of which each member is a part of x, or of y, or of both". In each case what seems to be meant is that a division can be made between such parts. It is not meant either that there is any actual division or separation or that there is not. It is conceivable that a division should be made at any line at which the razor edge might be placed. But this is not to say that as things are now the razor edge distinguishes what belongs to or is actually included in some thing from what does not belong to it.

Perhaps it will be said that the razor edge distinguishes at any rate what <u>would</u> belong to one part from what would belong to another if a division were actually made there. And it may be argued that it would be impossible to specify now what would belong to the right hand part rather than to the left | [52] hand part after such a division unless it were true now that what is so specified belongs to this part of the body and not to that part. When we say "this part might actually be divided from that part" we are speaking of something that is really there in the body and it is of this that we are saying that it might be separated; and we do not mean even partially the same by "this part" and "that part". This is the same argument as that which I have placed third in the list of objections cited above.

Now I agree that we may speak truly of a real part of the body as extending just as far as the razor edge and no further. I agree also that we may speak of a part of a single body as identical with what is later separated, in the sense that what has been separated does not contain any concrete volume more or any concrete volume less than what was referred to as that real part. But if when we referred to the part of the single body as extending just up to the razor edge and no further we had added, "This is a real part to which nothing on the other side of the razor edge belongs", then I think we should have been in danger of misleading. Perhaps we should be justified in saying that nothing on the other side of the razor edge belongs to it as a part. But we should be wrong if we suggested that there was a real boundary or surface there at which anything ended. The distinction of this part from the part on the other side of the razor edge is in a sense real enough. But it is not a real distinction in the sense in which a distinction between separate things is so. The part, while it remains a part of a single body, is not, so to speak, a thing in itself. By this I do not mean, of course, that it is in any sense identical with or reducible to anything else. I mean partly what can be expressed by saying that to say that what is to the right of the razor edge ends there as a part (in this sense of part of a single body) | [53] implies that it does not end there as a thing.

If this is so, then there is nothing in this precise distinction of parts from one another which conflicts with my contention that within a body there can be only inner surfaces, and that an inner surface can be a surface only of something which includes it. I said that an inner surface could for this reason not be said to be a beginning or ending of anything, and that we could not say that within a body there was any surface with which a part of the body began or ended. In this sense of "being a surface with which something ends" an inner surface could not be a surface with which a part ended even though a part were said to extend up to it and not beyond it. In this sense a part does not have any surface with which it ends, even though it may be said to be definitely limited in extend.

If this can be accepted, then I think that the remaining objection, the second of those cited above, can offer no real difficulties. This objection was to the effect that if there are two square pieces of paper of which one is larger than the other, then the larger one may be said to contain a part exactly equal to and of exactly the same dimensions as the smaller piece; and it was suggested that this means that the larger piece, although a single body, contains a part having definite boundaries. I agree that it may be said to contain a part having definite boundaries if this means nothing more than that it contains a part definitely limited in extent. But I deny that this implies that it contains a part to which an inner sur-

face can be said to belong without being included. I think, therefore, that the fact‹s› which might at first sight be mentioned as objections to the view I have suggested of the distinction between inner and outer surfaces are not really inconsistent with it. They would be so only if a definitely limited part were necessarily also a definitely limited thing. | [54] But I can see no reason to believe that this is the case, and there seem to me to be very serious difficulties for any view that were to maintain that it was.

I introduced this discussion of inner surfaces in connection with my attempt to show why I believe that what is meant by contact of bodies cannot be stated simply in terms of intersecting volumes. I said in the first place that it obviously could not be stated in terms of intersecting bodies. And I suggested that a statement about intersecting volumes within the common volume or area which two bodies filled would not explain what was meant by the separateness of two bodies from one another. I suggested that if nothing more were meant by the contact of two bodies than what could be stated in this way, then a blue block could not be said to "adjoin" a red block in any different sense from that in which the blue block plus one third of the red block could be said to adjoin the remaining two thirds of the red block. I said that this consequence was one which I should not like to accept because it seemed to disregard much of what is ordinarily meant by "being a single body". My contentions, that there are important differences between inner and outer surfaces, and that within a single body there are none but inner surfaces, were intended to make clearer what I meant by this.

I do not profess to have given any proof that inner surfaces, in the sense in which I have been using the expression, are actually given in experience, beyond the appeal to what is meant or implied by saying that a volume is given as endlessly divisible, or endlessly reducible from two opposite sides in the way I have described. I do not believe that anything more than this is required; though I am aware that my own statement of the analysis of what is so given is woefully obscure and incomplete. But if I have not made it clear that my analysis of what is so given is the right one, I hope at any rate that I may have made a little clearer what it is that I am contending | [55] when I say that we cannot talk about the side of any part within the red block in exactly the same sense as we can talk about the side of the red block which is in contact with the blue one. If I have done this, then I think I may have made a little clearer what I mean by speaking of integral surfaces altogether. If so, then it should not be very difficult to show what is meant by integral lines and integral points. And if I can do this, then I think I can answer more clearly the question as to whether there is any considerable analogy between the sense in which a line can be said to contain points which

make it up on the one hand and the sense in which a class or ordered series can be said to comprise terms which are members of it on the other.

Before I pass to this question, however, I must explain one further point in what I mean by saying that when two bodies are in contact the outer surface or side of one touches the outer surface or side of the other. If the outer surfaces are integral constituents of the bodies to which they belong and are said to touch one another, then this must mean something different from what is meant in saying that the two bodies touch one another. A surface is not something which has an outside and an inside, and we cannot say of surfaces that the outside of the one touches the outside of the other, as we do of bodies. Further, it seems evident that no surface can be "next to" another surface. We should not know what was meant by saying that this surface was next to that one. On the other hand, if two surfaces are separated, then there is no sense in saying that they are in contact or that they touch. Thus if they are in contact they can be neither separated nor next to one another. Consequently we must say, I think, that they coincide.

This is a statement about the positions of the surfaces in question. To say of two surfaces which are integral constituents of real things that they coincide or are in the same place sounds like a statement that two integral constituents | [56] of bodies interpenetrate. And this is likely to make one think that there is something wrong somewhere. But I think this would result from some misunderstanding of what can be meant by saying that two surfaces are in the same place. It clearly cannot mean that two surfaces interpenetrate, since there is no sense in speaking of a penetration of a surface by anything (unless it might be in speaking of penetration of a surface from one of its ends, – which could obviously have nothing to do with the case we are considering). I fancy, however, that it would also, and for similar reasons, be absurd to say of two surfaces that they excluded one another. On the other hand it seems plain that the side of a body is somewhere; and that when two bodies are in contact the side of the one is where the side of the other is. The difficulty, if there is one, is in seeing what the "is somewhere" means in such a connection.

When we say of anything that it "is somewhere" we generally have in mind the sort of thing that is meant in saying that a body is somewhere. When we say that the side of a body is somewhere we must mean something rather different; although the "somewhere" of a side or surface must obviously be connected in one way or another with the "somewhere" of a body.

The place of an outer surface cannot be a place in itself, so to speak. We might say that the place of the side of a body is the edge of the

place of the body. In this way it is as inconceivable without the place of a body as the side of a body is inconceivable without something of which it is the side. It is not a place which anything capable of being a thing by itself could conceivably occupy.

On the other hand it is the same place which may be thought of either as the edge of the place of a body in one direction or as the edge of the place of a body in another direction; or again, as the median of the place of a body which includes it. The same place may thus be at one time the | [57] place where an outer surface is, and at another time the place where an inner surface is. (It would obviously be just as absurd to try to think of the place where an inner surface is without thinking of the place of some body which includes it, as it is to try to think of the place of the side of a body without thinking of the place of the body.) There is a certain difference in what is meant by saying that a surface is there, according as we speak of an outer surface of a body in one direction, or of an outer surface of a body in another direction, or of an inner surface. The difference, however, is not in the meaning of "there", but rather in the way in which the surface in question is there. The expression "way" in such connections is vague, but I cannot think of any that is much more precise. It is tempting to say that the difference between the way in which an outer surface (in either direction) can be at the place in question on the one hand, and the way in which an inner surface can be there on the other is some sort of difference in the completeness of its being there. If a surface is at the place only as the side of a body which extends from there in one direction, then it is always possible for something to be there as the side of a body which extends in another direction. If an inner surface is at that place, however, then it is there in every way in which it is possible for anything to be there.

The reference to differences in the "completeness of a surface's being at a place" may seem objectionable. If it is there at all, it may be said, then it must be completely there; and to speak of anything as being "incompletely at that place" has no meaning. I do not wish to minimise the difficulties connected with this way of speaking. But I think they are difficulties about the choice of language, and do not constitute any fundamental objection to the point I wish to make.

The "differences in completeness" which I have in mind are connected with what I should like to call "differences in | [58] completeness" or even "quantitative differences" in the surfaces themselves. (By this I do not mean, of course, any quantitative differences in the two dimensional areas of the surfaces). It might be said that the connection of an outer surface with the body to which it belonged was in a sense less than that of an inner surface would be, simply because the outer surface was connected with or belonged to the body "only on one side", so to

speak, whereas an inner surface, being essentially included in a body, would belong to the body "on both its sides". To speak in this way of differences of more and less in a surface's belonging to a body is likely to be misleading, since it may suggest that one and the same surface could partly belong to a body and partly not belong to it. But in so far as a surface does not belong to a body it is not a surface at all; (at any rate it is not the sort of surface which we call a bodily surface. I have not discussed the question whether there are purely spatial surfaces which are not bodily surfaces, and what, if there are such, their differences from bodily surfaces may be. I do not see that it is necessary to do so in the present connection.) As a surface it is essentially "of a body". Its connection with the body is not only necessary to its existence, it is also part of what the surface is. And any reduction in its connection with the body must be a reduction in it as a surface.

Here again I am aware that my language is unhappy. To suggest with regard to anything that it is its connection with something else seems to be reminiscent of "aspects", "perspectives", "identity in difference", and so on. But I do not think my contention has anything to do with the views which have made such expressions familiar. What I want to suggest is that a surface is not something which has a relation to a body; or at any rate that it is misleading to | [59] put it in this way. It cannot properly be described as the bearer of a relation "in" which it "stands" to the body. It is better to say that it is itself something relative and essentially belongs to the body. But this does not mean that it is identical with that to which it "belongs". There are perhaps no terms which have been used with more confusion in philosophy than the terms "relative" and "absolute". I think that the confusion has been heightened, particularly in recent times, by the tendency apparent in some writings to take the so-called "relations of comparison" as the fundamental type of all relations. But to a large extent this tendency is supported by common language, and particularly by the common usage of such terms as "relative" and "relatively". If it be said that "size is relative" one of the things that this means is that statements as to the size of anything are statements of comparison, – that they are statements that the things in question are larger than or smaller than or equal to something else. And much the same can be said, I think, regarding the majority of statements to the effect that some conception or other is a "relative conception". I hope I need hardly remark that this is not at all the sense of my statement that a surface is something relative. And perhaps there is no excuse for my using language in a way so contrary to the ordinary usage. But I know of no other expression which will suggest at all what I mean by saying that a surface is essentially "of something" without being identical with it and without being a quality or property of it. It seems to me better to say

that a surface is itself something relative rather than to say that a surface is "essentially connected with" something else, chiefly because the type of "connection" to which we should wish to refer by such an expression would be just what we call "being a surface". The phrase "essential connection" is vague and is more likely to suggest something | [60] of a quite different sort, such as what is meant when we say that two properties are essentially connected in something if they mutually imply one another. Whether or not there is an "essential connection" of this sort between a surface and that of which it is a surface, – and perhaps there is, – at any rate this is not what I am speaking of when I say that a surface is something which is itself relative. Finally, I say that it is better to say that a surface is itself something relative than to suggest that it is its connection to something else, since the latter statement might be taken to imply that a surface is "a relation". I have said already that I do not wish to say this. I should agree that there is no justification for speaking of a relation unless we can say that it is "had" by something "to" something else. And one of the things I am most anxious to deny is that a surface, in the sense of "integral surface", is a relation which one body has to another.

It is partly for such reasons that I suggest that we cannot say that the connection of an outer surface with a body is less than the connection of an inner surface (of the same area) unless we mean that the surface itself is less. I have suggested that this might be called a "quantitative difference" between the surfaces, although it is obvious that the term "quantitative" would have a derivative meaning in this connection and could not be taken to mean that the surfaces were of different sizes. It would refer simply to a difference in the measure in which a surface was something relative, or, in other words, of the measure in which it was a surface. I say that this is a derivative sense of "quantitative" since it seems to me that the "quantity" of a surface (which in this connection, let me repeat, does not mean its two dimensional area) does stand in a definite relation to the extensive quantity of something else. The question of what this | [61] relation is is one which I should prefer not to raise in this place, and I hope there is no need for me to do so. This quantitative difference of surfaces is, I think, the kind of difference which Brentano called a difference in the "plerosis" of surfaces.[6] In this terminology we might say that an inner surface was a surface in complete plerosis, whereas an outer surface would be a surface in half plerosis. Brentano used the expression "differences of plerosis", however, to refer not only to what I have been calling quantitative differences of surfaces, but also to the difference between a "right hand" outer surface and a "left hand"

6. Compare Psychologie, II‹.› Band (Leipzig 1925) p. 261, also Kategorienlehre, p. 171.

outer surface. I think it is useful to have such a common term for both these sorts of difference, since it helps to emphasise certain of the most important features of that difference which I have called a quantitative one. If we do use this terminology, however, it will be necessary to emphasise that certain differences in plerosis are differences of quantity or "completeness", whereas others are not.

Now I have said that this difference in quantity between surfaces is connected with what I have called a difference in the completeness of a surface's being at a given place. The objections mentioned to any such expression are justified against what would probably be the most natural interpretation of it. It is nonsense to say of a surface that it might remain unchanged as a surface and yet be either more completely or less completely where it is, – at any rate if the surface is supposed to be the surface of a resting body in each case. But it is not obvious that it is nonsense to say that another surface might have been more completely at this place than this | [62] surface is, – to say, for instance, that if there were an inner surface where this outer surface is, it would be more completely there than the outer surface is. If in fact this i‹s› not nonsense, then that is because the question of what we mean by saying that a surface, in distinction from a body, is at a place, is largely answered when we know what is meant by a surface in distinction from a body at all; and further, because one surface may be quantitatively less than another. I will try to explain this.

I do not wish to raise the question of what is meant by a body's being at a place. I am concerned simply with the distinction between a surface's being at a place and a body's being at a place. In order to know what is meant by a surface's being at a place one must know what is meant by a body's being at a place. But I am simply going to assume that this is known. I say then that the distinction between the location[7] of the surface and the location of that of which it is a surface is given as soon as we have distinguished the surface from that to which it belongs at all. This seems just as obviously true as it is that the distinction between the location of one part having volume and the location of another part having volume (in the same body) is given as soon as we have distinguished between the one part and the other, – although of course the location of a surface is something rather different from the location of a part having volume, just as the distinction of a surface from that of which it is a surface is different from the distinction of one part having volume from another. This may seem so trivial as to be hardly

7. I want to use "the location of a surface" to mean the same as "a surface's being at a place".

any explanation at all. But I cannot see that very much more in the way of explanation of this distinction is needed. | [63]

I have said that surfaces may belong to bodies in different ways, or, in other words, that they may be surfaces in different plerosis. I have just been speaking of the distinction between a surface and that to which it belongs, as far as this is relevant to what we mean by a surface's being at a place. And it is fairly clear that where surfaces are of different plerosis this distinction will be of a slightly different sort. This is clearest if we compare an inner surface with an outer surface in this respect. Suppose, for instance, that a body expands whilst its centre remains fixed. Then when it has expanded it will have an inner surface at the place at which one of its outer surfaces was before. But the inner surface's being at that place will differ from the outer surface's being at that place, and the difference will be a consequence of the difference in plerosis or the quantitative difference of the surfaces. A surface is a surface at all only in its connection with its body, and it is only in its connection with its body, or only as a surface of its body, that it can be anywhere. This peculiar "relativity" of a surface involves a corresponding "relativity" of its location. The connection of an outer surface with its body does not make it possible for the surface to be at the place where it is "from both sides", so to speak. Its location is "of" or "belongs to" the location of the body. And its location "belongs to" the location of its body in a different way from that in which the location of an inner surface does. If the body is continuous on both sides of the place in question and does not merely have an outer surface there, then there is an inner surface which is at that place "from both sides". The surface's being at that place "belongs to" or is "of" the location of the body at a place which includes that place. (It includes it, of course, not in the way in which one area may include another, but in the way in which anything having volume may include an inner surface.) I have suggested that | [64] the relation of the location of a surface to the location of its body is the same as the relation of the surface to the body (though I have also said that the expression "relation of a surface to its body" is likely to be misleading). And for this reason I have suggested that the inner surface's being at that place is more complete than the outer surface's being at that place could be.

I introduced this discussion of the completeness of a surface's being at a given place in order to try to make clearer what is meant by the coincidence of the outer surfaces of two bodies in contact. I said that part of what this means is that a side of the one body is where a side of the other body is. It seemed, however, that there might be an objection to saying this if one were going to hold that the side of a body were something real and integral to it, since the sides or outer surfaces of the two bodies could

not be identical and could not therefore be said to be in the same place. I suggested that this objection could be based only on a misunderstanding of what is meant by a surface's being at a place. I have tried to show in some measure what this does mean. And I hope it is clear from what I have said that if we speak of two surfaces as being in the same place we mean something very different from what would be meant by saying that two bodies were in the same place. If so much is clear, then the fact that it is impossible for two bodies to be in the same place is largely irrelevant to the question whether two surfaces can be at the same place.

The mere removal of this objection does not show positively that two surfaces can be at the same place, of course. But I think that what has been said in the course of the discussion does do something to show that they can. The only surfaces which come in question are outer surfaces, and outer surfaces of which one is a surface in the opposite direction or | [65] in opposite plerosis to the other. Now if an inner surface might have been at the place where one of these outer surfaces is, and if the outer surface is less completely at that place than an inner surface would be, then whilst the outer surface is there it should be possible for something else to be at that place in a way that is complementary, so to speak, to the way in which this outer surface is there. It would not be complementary to the surface or to its location, since it would not make the location of the outer surface any more nearly that of an inner surface than it would otherwise be. But it would be complementary as far as the "filling" of the place was concerned. The "something else" which might thus be at the place of the outer surface first mentioned would have to be a surface itself, of course. For the place of which we are speaking cannot be the place of anything but a surface. And if this surface were there only in the way complementary to that of the first mentioned surface, it would obviously have to be an outer surface. It would then be as impossible to say of the two surfaces that they were separated or in any sense at different places as it would be to say of an inner surface that it contained part-surfaces at different places. On the other hand, their coincidence at that place would be something very different from the location of an inner surface at that place. And this would be largely a consequence of the fact that the location of each of the outer surfaces there "belonged to" the location of a different body.

I do not wish to say anything more at this juncture about the outer surfaces of bodies in contact, though it is a matter that will be found to be fundamental to many of the questions I have to discuss later. I maintain that there is no objection to saying that two surfaces can be in the same place; and I suggest that the fact that outer surfaces are quantitatively less, or of less complete plerosis, than inner surfaces makes | [66] it positively clear that they can. I think therefore that there is no objection

to saying that when two bodies are in contact a side of the one coincides with a side of the other. I have gone into this discussion of contact partly to remove a possible objection to the view that the outer surfaces of bodies are real and integral to the bodies to which they belong. But I have also had another reason. My contention has been that the contact of different bodies cannot be analysed simply in terms of intersecting volumes. My discussion of coincidence has done nothing to show that it cannot, but has rather proceeded on the assumption that it cannot. But since I had contended that no analysis could be adequate that did not recognise integral surfaces, I felt that I ought to try to indicate the sort of statement that could be made in terms of integral surfaces. My general aim throughout has been to make clearer the notion of integral surfaces altogether.

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